

CUT & GLUE SORT

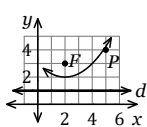


PARABOLAS: FOCUS AND DIRECTRIX

On the Parabola or Not? The Focus and the Directrix

Twelve points · coordinates, distances, drawings · three rows

FREE

1 The focus is $F(1, 4)$ and the directrix d is the line $y = 0$. P is the point $(9, 10)$.	2  $F(2, 3)$, d is $y = 1$, and P is $(5, 4)$. The curve is the parabola.	3 The focus is $F(0, 3)$ and the directrix d is the line $y = -3$. P is 6 from F, and 6 from the point $(6, -3)$, which is on d.
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Row 1. P is on the parabola.

Row 2. P is not on the parabola.

Row 3. Cannot tell. The piece does not give enough to decide.

1. **Hand out pages 3 and 4**, one of each per student, with scissors and glue. No calculator is needed.
2. **Say this once:** *“P is on the parabola when it is as far from the focus F as from the directrix d. The distance to d is the perpendicular one. A segment to d with no right-angle mark is slanted. A slanted segment is always longer than the perpendicular one. If the piece does not give enough to decide, put it in row 3.”*
3. **Check a sheet in ten seconds:** every row gets four pieces.

Student sheets	pages 3 and 4, one of each per student. Page 3 has the three rows. Page 4 has the pieces and a box that shows a parabola with F, d, V and one point, and says what the distance to d is.
Answers	page 5.

A point is on a parabola when its distance to the focus F equals its distance to the directrix d. The distance to d is always the perpendicular one. When d is a horizontal line $y = c$, it is the difference of the y-coordinates. When d is a vertical line $x = c$, it is the difference of the x-coordinates. Comparing the squares of the two distances keeps every number whole.

Row 2 holds the pieces that tempt students most. One point sits beside the drawn curve and looks as if it is on it. One is the focus itself. One drawing has a slanted segment to d as long as PF. One has a vertical directrix, where a student who takes the difference of the y-coordinates gets the wrong distance.

Row 3 holds four pieces that do not settle the question: the distance to F alone, a distance to some point of d that need not be straight below P, the x-coordinate of P alone, and a slanted segment longer than PF. The key gives two points for each, one on the parabola and one not, that both fit the piece. Guessing does not work: a simple guessing rule, such as sorting by whether the piece is a drawing, places at most six pieces correctly.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.

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Cut & Glue Sort — on the parabola or not?

Focus and directrix

Name _____

Period _____

Date _____

Each piece on the next page tells you about a point P and a parabola with focus F and directrix d. **Is P on the parabola? Glue the piece in its row.** Use only what the piece gives. Do not measure a drawing. Use its numbers.

A segment to d with no right-angle mark is slanted. The distance to d is shorter than it.

Row 1. P is on the parabola.

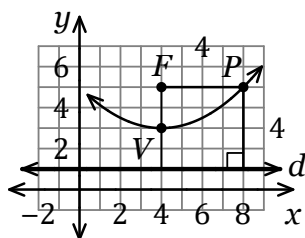
Row 2. P is not on the parabola.

Row 3. Cannot tell. The piece does not give enough to decide.

Cut & Glue Sort — the pieces

Cut along the dashed lines

Cut out the twelve pieces. Is P on the parabola? Glue each piece in its row on the other sheet. Use only what the piece gives. Do not measure a drawing. Use its numbers.

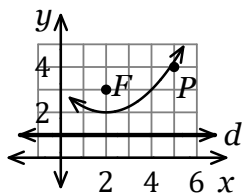


A point is on the parabola when its distance to the focus F equals its distance to the directrix d.
The distance to d is the perpendicular segment from the point to d. When d is the line $y = c$, it is the difference of the y-coordinates. When d is the line $x = c$, it is the difference of the x-coordinates.
A segment to d with no right-angle mark is slanted. The distance to d is shorter than it.
A drawing never decides. A point can look as if it is on the curve and miss it. Use the numbers.

1

The focus is $F(1, 4)$ and the directrix d is the line $y = 0$. P is the point $(9, 10)$.

2



$F(2, 3)$, d is $y = 1$, and P is $(5, 4)$. The curve is the parabola.

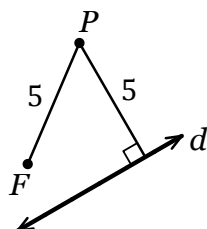
3

The focus is $F(0, 3)$ and the directrix d is the line $y = -3$. P is 6 from F, and 6 from the point $(6, -3)$, which is on d.

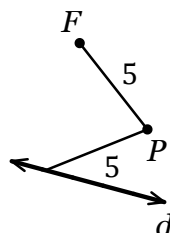
4

The focus is $F(-2, 1)$ and the directrix d is the line $y = -3$. P has x-coordinate 2. Its y-coordinate is not given.

5



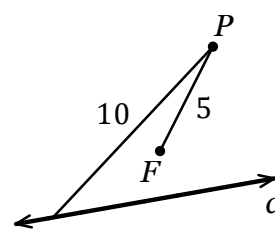
6



7

The focus is $F(2, 3)$ and the directrix d is the line $y = -1$. P is the focus F itself, so P is 0 from F.

8



9

The focus is $F(5, 1)$ and the directrix d is the line $x = -1$. P is the point $(5, 7)$.

10

The focus is $F(0, 1)$ and the directrix d is the line $y = -1$. P is 5 from F.

11

The focus is $F(0, 2)$ and the directrix d is the line $y = -2$. P is 4 from F, and 4 from d along the perpendicular.

12

The focus is $F(4, 0)$ and the directrix d is the line $x = 0$. P is the point $(4, 3)$.



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Each row holds four pieces. Every piece is described again here, with the reason. On this sheet the distance from a point to a line is the length of the perpendicular segment.

1 **P is on the parabola.**

1. The focus is $F(1, 4)$ and the directrix d is the line $y = 0$. P is the point $(9, 10)$. *Why:* $PF^2 = 64 + 36 = 100$. d is horizontal, so the distance to d is the difference of the y -coordinates, which is 10. Its square is 100. The two squares are equal, so P is on the parabola.

5. A drawing: PF is marked 5, and a segment from P to d is marked 5, with a right-angle mark. *Why:* $PF = 5$, and the perpendicular distance to d is 5. The two are equal, so P is on the parabola.

9. The focus is $F(5, 1)$ and the directrix d is the line $x = -1$. P is the point $(5, 7)$. *Why:* $PF^2 = 0 + 36 = 36$. d is vertical, so the distance to d is the difference of the x -coordinates, which is 6. Its square is 36. The two squares are equal, so P is on the parabola.

11. The focus is $F(0, 2)$ and the directrix d is the line $y = -2$. P is 4 from F , and 4 from d along the perpendicular. *Why:* P is 4 from F and 4 from d , measured along the perpendicular. The two distances are equal, so P is on the parabola.

2 **P is not on the parabola.**

2. The focus is $F(2, 3)$ and the directrix d is the line $y = 1$. P is the point $(5, 4)$. The drawing shows the parabola. *Why:* $PF^2 = 9 + 1 = 10$. d is horizontal, so the distance to d is the difference of the y -coordinates, which is 3. Its square is 9. The two squares differ, so P is not on the parabola. It looks as if it is on the curve, but it is just below it.

6. A drawing: PF is marked 5, and a segment from P to d is marked 5, slanted, with no right-angle mark. *Why:* The slanted segment is not the distance to d . The perpendicular is the shortest segment from P to d , so it is shorter than the slanted 5. P is nearer d than F , so it is not on the parabola.

7. The focus is $F(2, 3)$ and the directrix d is the line $y = -1$. P is the focus F itself, so P is 0 from F . *Why:* P is 0 from F but more than 0 from d , because F is not on d . The focus is never on the parabola.

12. The focus is $F(4, 0)$ and the directrix d is the line $x = 0$. P is the point $(4, 3)$. *Why:* $PF^2 = 0 + 9 = 9$. d is vertical, so the distance to d is the difference of the x -coordinates, which is 4. Its square is 16. The two squares differ, so P is not on the parabola. A student who uses P 's y -coordinate, 3, in place of its x -coordinate, 4, gets 3, the same as PF , and wrongly puts P on the parabola.

3 **Cannot tell. The piece does not give enough to decide.**

3. The focus is $F(0, 3)$ and the directrix d is the line $y = -3$. P is 6 from F , and 6 from the point $(6, -3)$, which is on d . *Why:* The point $(6, -3)$ need not be straight below P , so 6 is not known to be the distance to d . For example, $(6, 3)$ fits the piece and is on the parabola, and $(0, -3)$ fits it and is not.

4. The focus is $F(-2, 1)$ and the directrix d is the line $y = -3$. P has x -coordinate 2. Its y -coordinate is not given. *Why:* Only one point of the curve has $x = 2$. P could be that point or any other point with that x . For example, $(2, 1)$ is on the parabola and $(2, 0)$ is not.

8. A drawing: PF is marked 5, and a segment from P to d is marked 10, slanted, with no right-angle mark. *Why:* The slanted segment is not the distance to d . The perpendicular is shorter than 10, but the piece does not say how long it is. If it is 5, P is on the parabola. If it is 6, P is not. Both fit the numbers, so the piece cannot be placed.

10. The focus is $F(0, 1)$ and the directrix d is the line $y = -1$. P is 5 from F . *Why:* Only the distance to F is given. The distance to d is not. For example, $(-4, 4)$ fits the piece and is on the parabola, and $(-5, 1)$ fits it and is not.

To check a piece with a student, ask: “Do you know the distance to F ? Do you know the perpendicular distance to d ? Are they equal?”