

ANSWER MAZE



PARABOLAS: FOCUS AND DIRECTRIX

The Equation of a Parabola from Its Focus and Directrix

Vertex, p and $(x - h)^2 = 4p(y - k)$ · one page · 10–15 minutes · nothing to prepare

FREE

5 **F(2, 5). The directrix d is $y = -1$.**

$$(x - 2)^2 = 24(y - 2) \rightarrow \text{go to box 3}$$

$$(x - 2)^2 = 12(y - 2) \rightarrow \text{go to box 2}$$

Hint: Find V halfway between F and d . Then p is the distance from V to F .

2 **F(-1, -4). The directrix d is $y = 2$.**

$$(x + 1)^2 = -12(y + 4) \rightarrow \text{go to box 6}$$

$$(x + 1)^2 = -12(y + 1) \rightarrow \text{go to box 7}$$

Hint: F is inside the curve. Find the point of the curve halfway to d .

1. **Hand out page 3**, one per student, and scrap paper. No calculator is needed.
2. **Say this once:** *“Start at box 5. Each box gives a focus F and a directrix d . Find the vertex halfway between them, then p . Pick the equation and go to the box it names. Write every box number in the row at the bottom.”*
3. **Check a sheet quickly.** A student with no mistakes writes **5 2 7 4 8 1**: six boxes, then out.

Student sheet	page 3, one per student. A box at the top shows a parabola with F , d , V and one point, and says what the vertex, p and the equation are.
Answers	page 4.

A point is on a parabola when it is as far from the focus F as from the directrix d . The vertex V is halfway between F and d . The number p is the distance from V to F , positive when F is above V and negative when F is below V . With $V = (h, k)$, squaring “distance to F = distance to d ” gives $(x - h)^2 = 4p(y - k)$. Every directrix on this sheet is a horizontal line.

Each wrong equation comes from one of four mistakes. Taking p as the whole distance from F to d doubles the number in front (two boxes). Giving p the wrong sign turns the curve toward d (two boxes). Writing $1/(4p)$ in front instead of $4p$ gives a fraction (one box). Putting the vertex at the focus changes the bracket with y (three boxes). A wrong number in front sends the student to box 3, and a wrong vertex to box 6. Both of those boxes ask the same kind of question again, and both of their answers lead back to the route.

The row at the bottom is the check. A clean run is six numbers. Each wrong answer adds at least one more, and the extra box number tells you which mistake it was. Guessing does not get a student out: every simple guessing rule, such as always taking the larger number in front, goes wrong within the first four boxes.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Answer Maze — the equation of a parabola

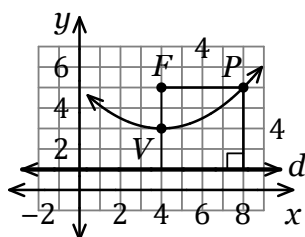
Start at box 5

Name _____

Period _____

Date _____

Start at box 5. **Each box gives a focus F and a directrix d. Pick the equation of the parabola.** Find V and p first. Then go to the box your answer names. Write each box number in the row at the bottom as you go.



A point is on the parabola when its distance to the focus F equals its distance to the directrix d. **The distance to d** is the perpendicular segment to d. Here d is a line $y = c$, so it is the difference of the y-coordinates. **The vertex V** is halfway between F and d, on the perpendicular from F to d. V is on the curve. F is not. **p** is the distance from V to F, with a sign. F above V: p is positive and the curve opens up. F below V: p is negative and it opens down. The curve opens away from d. **The equation** with vertex $V(h, k)$ is $(x - h)^2 = 4p(y - k)$. It comes from squaring “distance to F = distance to d”.

1 F(-3, 2). The directrix d is $y = -4$.

$$(x + 3)^2 = 12(y + 1) \rightarrow \text{you are out}$$

$$(x + 3)^2 = 12(y - 2) \rightarrow \text{go to box 6}$$

Hint: The vertex is halfway between F and d. Check the bracket with y.

2 F(-1, -4). The directrix d is $y = 2$.

$$(x + 1)^2 = -12(y + 4) \rightarrow \text{go to box 6}$$

$$(x + 1)^2 = -12(y + 1) \rightarrow \text{go to box 7}$$

Hint: F is inside the curve. Find the point of the curve halfway to d.

3 F(4, 3). The directrix d is $y = -1$.

$$(x - 4)^2 = -8(y - 1) \rightarrow \text{go to box 5}$$

$$(x - 4)^2 = 8(y - 1) \rightarrow \text{go to box 2}$$

Hint: p has a size and a sign. Find both by going from V to F.

4 F(-2, 4). The directrix d is $y = -2$.

$$(x + 2)^2 = 12(y - 1) \rightarrow \text{go to box 8}$$

$$(x + 2)^2 = (1/12)(y - 1) \rightarrow \text{go to box 3}$$

Hint: Find p first. Then compare each answer with the equation in the box at the top.

5 F(2, 5). The directrix d is $y = -1$.

$$(x - 2)^2 = 24(y - 2) \rightarrow \text{go to box 3}$$

$$(x - 2)^2 = 12(y - 2) \rightarrow \text{go to box 2}$$

Hint: Find V halfway between F and d. Then p is the distance from V to F.

6 F(-4, -3). The directrix d is $y = -1$.

$$(x + 4)^2 = -4(y + 2) \rightarrow \text{go to box 7}$$

$$(x + 4)^2 = -4(y + 3) \rightarrow \text{go to box 2}$$

Hint: V is the point of the curve between F and d. Find it first.

7 F(0, -2). The directrix d is $y = 2$.

$$x^2 = 8y \rightarrow \text{go to box 3}$$

$$x^2 = -8y \rightarrow \text{go to box 4}$$

Hint: Is F above d or below it? The curve opens toward F.

8 F(1, 2). The directrix d is $y = 4$.

$$(x - 1)^2 = -8(y - 3) \rightarrow \text{go to box 3}$$

$$(x - 1)^2 = -4(y - 3) \rightarrow \text{go to box 1}$$

Hint: Find V first. Then check the size and the sign of p.

Write every box number here, in order.

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The route, in order. Boxes 3 and 6 are not on it. A student reaches them only by a wrong answer. They are the last two rows of the table. This sheet writes the equation as $(x - h)^2 = 4p(y - k)$, with $V = (h, k)$ and p the distance from V to F , negative when F is below V . Some textbooks use other forms, so tell the class which one this sheet uses.

5 → 2 → 7 → 4 → 8 → 1 → out

	The box's question	Right answer, and why	Wrong answer, and why a student picks it
1	Box 5 F(2, 5), and d is the line $y = -1$. Pick the equation of the parabola.	$(x - 2)^2 = 12(y - 2)$. V(2, 2) is halfway between F(2, 5) and d: $y = -1$. F is above V, so $p = 3$ and $4p = 12$. The equation is $(x - 2)^2 = 12(y - 2)$.	" $(x - 2)^2 = 24(y - 2)$ " goes to box 3. p is the distance from V to F, which is half the distance from F to d. A student who takes the whole distance, 6, gets 24 in front.
2	Box 2 F(-1, -4), and d is the line $y = 2$. Pick the equation of the parabola.	$(x + 1)^2 = -12(y + 1)$. V(-1, -1) is halfway between F(-1, -4) and d: $y = 2$. F is below V, so $p = -3$ and $4p = -12$. The equation is $(x + 1)^2 = -12(y + 1)$.	" $(x + 1)^2 = -12(y + 4)$ " goes to box 6. The vertex is halfway between F and d, not at F. A student who puts the vertex at F(-1, -4) writes the bracket with y using -4 instead of -1.
3	Box 7 F(0, -2), and d is the line $y = 2$. Pick the equation of the parabola.	$x^2 = -8y$. V(0, 0) is halfway between F(0, -2) and d: $y = 2$. F is below V, so $p = -2$ and $4p = -8$. The equation is $x^2 = -8y$.	" $x^2 = 8y$ " goes to box 3. F is below d, so the curve opens down and p is negative. A student who gets the sign wrong writes 8 in front.
4	Box 4 F(-2, 4), and d is the line $y = -2$. Pick the equation of the parabola.	$(x + 2)^2 = 12(y - 1)$. V(-2, 1) is halfway between F(-2, 4) and d: $y = -2$. F is above V, so $p = 3$ and $4p = 12$. The equation is $(x + 2)^2 = 12(y - 1)$.	" $(x + 2)^2 = (1/12)(y - 1)$ " goes to box 3. The number in front of the bracket with y is $4p = 12$. A student who writes $1/(4p)$ there gets $(x + 2)^2 = (1/12)(y - 1)$.
5	Box 8 F(1, 2), and d is the line $y = 4$. Pick the equation of the parabola.	$(x - 1)^2 = -4(y - 3)$. V(1, 3) is halfway between F(1, 2) and d: $y = 4$. F is below V, so $p = -1$ and $4p = -4$. The equation is $(x - 1)^2 = -4(y - 3)$.	" $(x - 1)^2 = -8(y - 3)$ " goes to box 3. p is the distance from V to F, which is half the distance from F to d. A student who takes the whole distance, 2, gets -8 in front.
6	Box 1 F(-3, 2), and d is the line $y = -4$. Pick the equation of the parabola.	$(x + 3)^2 = 12(y + 1)$. V(-3, -1) is halfway between F(-3, 2) and d: $y = -4$. F is above V, so $p = 3$ and $4p = 12$. The equation is $(x + 3)^2 = 12(y + 1)$.	" $(x + 3)^2 = 12(y - 2)$ " goes to box 6. The vertex is halfway between F and d, not at F. A student who puts the vertex at F(-3, 2) writes the bracket with y using 2 instead of -1.
	Box 3 F(4, 3), and d is the line $y = -1$. Pick the equation of the parabola.	$(x - 4)^2 = 8(y - 1)$. V(4, 1) is halfway between F(4, 3) and d: $y = -1$. F is above V, so $p = 2$ and $4p = 8$. The equation is $(x - 4)^2 = 8(y - 1)$.	" $(x - 4)^2 = -8(y - 1)$ " goes to box 5. F is above d, so the curve opens up and p is positive. A student who gets the sign wrong writes -8 in front.
	Box 6 F(-4, -3), and d is the line $y = -1$. Pick the equation of the parabola.	$(x + 4)^2 = -4(y + 2)$. V(-4, -2) is halfway between F(-4, -3) and d: $y = -1$. F is below V, so $p = -1$ and $4p = -4$. The equation is $(x + 4)^2 = -4(y + 2)$.	" $(x + 4)^2 = -4(y + 3)$ " goes to box 2. The vertex is halfway between F and d, not at F. A student who puts the vertex at F(-4, -3) writes the bracket with y using -3 instead of -2.

How to check any box. Find V halfway between F and d. Count p from V to F, with a minus sign when F is below V. Then write $(x - h)^2 = 4p(y - k)$. A point of the answer, such as V itself, can be checked: it must be as far from F as from d.

A student can find the last four boxes of the route by following the arrows backward from "you are out." So ask a student with a clean row to explain one box.