

MYSTERY PICTURE

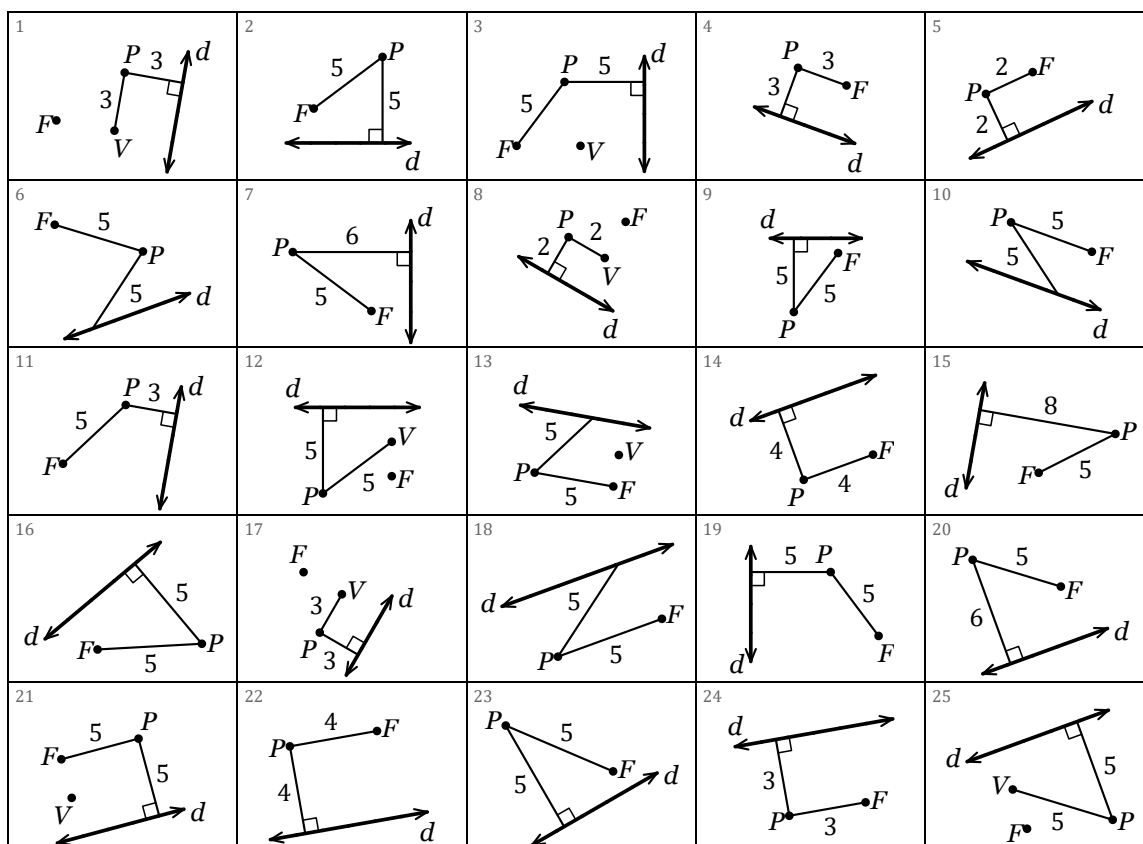


PARABOLAS: FOCUS AND DIRECTRIX

As Far from the Focus as from the
Directrix?

25 drawings · the focus, the directrix and the perpendicular · nothing to
calculate · a picture appears

FREE



Mystery Picture — how to run it

25–35 minutes

1. **Give each student page 3** and a pencil. Nothing is calculated, and nothing is measured.
2. **Say this once:** *“A point is on the parabola when it is as far from F as from d . Do the two numbers show that? Fill the square with slanted lines if they do. Leave it white if they do not.”*
3. **Let them work.** The picture shows when they are right. Nothing else is needed.

Student sheet	page 3, one per student. A box at the top shows a parabola with F , d , V and one point, and says what the distance to d is.
Answers	page 4, for you.

Every square shows a directrix d , a focus F , a point P and two segments from P , each marked with its length. The two numbers prove that P is on the parabola only when one segment goes to F , the other meets d at a right angle, and the two numbers are equal. The parabola itself is not drawn. Twelve squares are filled and thirteen are white.

Each white square leaves out one of those three checks. In five the first segment goes to the vertex V instead of the focus. In four the segment to d is slanted, so it is not the distance to d . One of them, square 6, goes to the point of d straight below F , a very common mistake. In four the segments are right but the numbers are different. In every white square P really is off the parabola.

If part of a student's picture looks wrong, ask about that square: *“Which segment is the distance to F , and which is the distance to d ?”* Guessing does not work. Every simple guessing rule, such as filling every square whose two numbers are equal, gets at least eight squares wrong. A student who skips one of the three checks gets four to five squares wrong, and the picture comes out broken.



This is the free lesson this activity came from. The worksheet, the full answers and the lesson plan are on it, at three levels of difficulty.
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Mystery Picture — is P as far from F as from d?

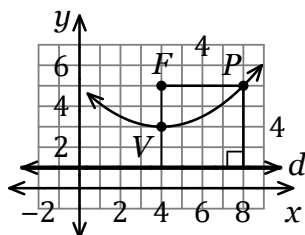
Focus and directrix

Name _____

Period _____

Date _____

Each square shows a directrix d , a focus F , a point P and two segments from P with their lengths. **Use only the printed numbers. Do not measure.** If the two lengths show that P is as far from F as from d , fill the square with slanted lines. If they do not show it, leave the square white. A picture appears. If part of it is missing, or one square does not fit, check that square again.



A point is on the parabola when its distance to the focus F equals its distance to the directrix d .
The distance to d is the segment from the point that meets d at a right angle. It carries the right-angle mark. Any slanted segment to d is longer.
The vertex V is halfway between F and d , on the perpendicular from F to d . V is on the curve. F is not.

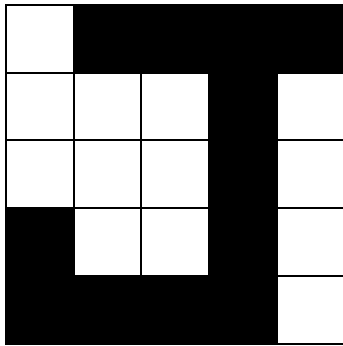
1 	2 	3 	4 	5
6 	7 	8 	9 	10
11 	12 	13 	14 	15
16 	17 	18 	19 	20
21 	22 	23 	24 	25



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Mystery Picture — answers

For the teacher



The finished picture is **the letter J**. Twelve squares are filled.

A square is filled when its two numbers prove that P is as far from F as from d: one segment to F, one segment to d with a right-angle mark, and equal numbers. The distance from a point to a line is always the perpendicular distance.

The square	Answer	Why
1 PV = 3. P to d = 3, with a right-angle mark.	white	The segment goes to the vertex V, not to F. In fact $PF^2 = 18$, not 9, so P is not on the parabola.
2 PF = 5. P to d = 5, with a right-angle mark.	fill	Both distances are 5, and the segment to d is perpendicular, so P is on the parabola.
3 PF = 5. P to d = 5, with a right-angle mark.	fill	Both distances are 5, and the segment to d is perpendicular, so P is on the parabola.
4 PF = 3. P to d = 3, with a right-angle mark.	fill	Both distances are 3, and the segment to d is perpendicular, so P is on the parabola.
5 PF = 2. P to d = 2, with a right-angle mark.	fill	Both distances are 2, and the segment to d is perpendicular, so P is on the parabola.
6 PF = 5. P to d = 5, slanted, with no right-angle mark.	white	A slanted segment is not the distance to d. The true distance is 3, less than $PF = 5$, so P is not on the parabola.
7 PF = 5. P to d = 6, with a right-angle mark.	white	P is 5 from F but 6 from d, so P is not on the parabola.
8 PV = 2. P to d = 2, with a right-angle mark.	white	The segment goes to the vertex V, not to F. In fact $PF^2 = 8$, not 4, so P is not on the parabola.
9 PF = 5. P to d = 5, with a right-angle mark.	fill	Both distances are 5, and the segment to d is perpendicular, so P is on the parabola.
10 PF = 5. P to d = 5, slanted, with no right-angle mark.	white	A slanted segment is not the distance to d. The true distance is 3, less than $PF = 5$, so P is not on the parabola.
11 PF = 5. P to d = 3, with a right-angle mark.	white	P is 5 from F but 3 from d, so P is not on the parabola.
12 PV = 5. P to d = 5, with a right-angle mark.	white	The segment goes to the vertex V, not to F. In fact $PF^2 = 17$, not 25, so P is not on the parabola.
13 PF = 5. P to d = 5, slanted, with no right-angle mark.	white	A slanted segment is not the distance to d. The true distance is 4, less than $PF = 5$, so P is not on the parabola.
14 PF = 4. P to d = 4, with a right-angle mark.	fill	Both distances are 4, and the segment to d is perpendicular, so P is on the parabola.
15 PF = 5. P to d = 8, with a right-angle mark.	white	P is 5 from F but 8 from d, so P is not on the parabola.
16 PF = 5. P to d = 5, with a right-angle mark.	fill	Both distances are 5, and the segment to d is perpendicular, so P is on the parabola.
17 PV = 3. P to d = 3, with a right-angle mark.	white	The segment goes to the vertex V, not to F. In fact $PF^2 = 18$, not 9, so P is not on the parabola.
18 PF = 5. P to d = 5, slanted, with no right-angle mark.	white	A slanted segment is not the distance to d. The true distance is 3, less than $PF = 5$, so P is not on the parabola.
19 PF = 5. P to d = 5, with a right-angle mark.	fill	Both distances are 5, and the segment to d is perpendicular, so P is on the parabola.
20 PF = 5. P to d = 6, with a right-angle mark.	white	P is 5 from F but 6 from d, so P is not on the parabola.
21 PF = 5. P to d = 5, with a right-angle mark.	fill	Both distances are 5, and the segment to d is perpendicular, so P is on the parabola.
22 PF = 4. P to d = 4, with a right-angle mark.	fill	Both distances are 4, and the segment to d is perpendicular, so P is on the parabola.
23 PF = 5. P to d = 5, with a right-angle mark.	fill	Both distances are 5, and the segment to d is perpendicular, so P is on the parabola.
24 PF = 3. P to d = 3, with a right-angle mark.	fill	Both distances are 3, and the segment to d is perpendicular, so P is on the parabola.
25 PV = 5. P to d = 5, with a right-angle mark.	white	The segment goes to the vertex V, not to F. In fact $PF^2 = 17$, not 25, so P is not on the parabola.