

SCAVENGER HUNT



PARABOLAS: FOCUS AND DIRECTRIX

Points on a Parabola: From the Focus and the Directrix

Vertex, p and the equation · checked by the two distances · the cards check the answers

FREE

ANSWER

—23

A parabola has focus $F(-3, -3)$ and directrix $y = 1$.

The point $(9, ?)$ is on it.

Find its y -coordinate.

The vertex is on the curve. The focus is not on the curve.

B

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1. **Tape the 16 cards around the room**, in any order. Each card has a large letter in its bottom left corner. Start each group at a *different* one of these eight cards: *B W S C G R U L*.
2. **Say this once:** *“Each card gives a focus, a directrix and an x . Find the vertex halfway between them, then p . Put x into the equation and find y . Check that your point is as far from F as from d . Then walk to the card with your y at the top.”*
3. **Each group writes one line per card.** Eight right answers bring the group back to its first card.

16 cards	pages 3–10, one copy. There are two cards on each sheet. Cut the whole stack once.
Record sheet	page 11, one per student. It shows a parabola with its focus, directrix and vertex, the steps, and a worked example.
Answers	pages 12–13, for you only. Do not put them up in the room.

A point is on a parabola when it is as far from the focus F as from the directrix d . The vertex V is halfway between F and d . The number p is the distance from V to F , positive when F is above V and negative when F is below. With $V = (h, k)$, the equation is $(x - h)^2 = 4p(y - k)$. For example, $F(2, 3)$ and $d: y = -1$ give $V(2, 1)$ and $p = 2$; at $x = 6$ the equation gives $y = 3$, and the point $(6, 3)$ is 4 from F (the distance formula: $\sqrt{(4^2 + 0^2)} = 4$) and 4 from d (the difference of the y -coordinates: $3 - (-1) = 4$). Every answer on the cards can be checked that way before the group walks.

Eight of the sixteen cards make the ring. A group reaches the other eight only by a wrong answer, but all the cards look the same. Two mistakes take turns around the ring. Some students take p to be the whole distance from F to d instead of half of it. Others give p the wrong sign, so their curve opens toward d . Each mistake lands on a card off the ring, and that card sends the group back to the card it got wrong. Eight of the cards have the focus below the directrix, so their curves open down.

The check is the length of the group’s list. A clean run is **eight lines**. Each wrong answer adds two. Every card has a line in italics under its question. It tells a group that arrived by a wrong answer what to check. A group that arrived by a right answer skips it. Guessing does not work: no single rule, such as walking to the nearest number, gets more than two of the cards.



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ANSWER

—8

A parabola has focus $F(-5, -5)$ and directrix $y = -7$.

The point $(5, ?)$ is on it.

Find its y-coordinate.

Is p the distance from V to F ? It is half the distance from F to d .

A

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ANSWER

—23

A parabola has focus $F(-3, -3)$ and directrix $y = 1$.

The point $(9, ?)$ is on it.

Find its y-coordinate.

The vertex is on the curve. The focus is not on the curve.

B

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ANSWER

23

A parabola has focus $F(-4, -1)$ and directrix $y = -5$.

The point $(4, ?)$ is on it.

Find its y-coordinate.

Subtracting a negative number adds it. Check each minus sign.

C

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ANSWER

-10

A parabola has focus $F(3, -7)$ and directrix $y = -3$.

The point $(-9, ?)$ is on it.

Find its y-coordinate.

p is half the distance from F to d , not the whole distance.

E

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ANSWER

5

A parabola has focus $F(3, 9)$ and directrix $y = 7$.

The point $(7, ?)$ is on it.

Find its y-coordinate.

The curve opens toward F . Is your point on the side of F ?

G

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ANSWER

27

A parabola has focus $F(3, -8)$ and directrix $y = -6$.

The point $(9, ?)$ is on it.

Find its y-coordinate.

A curve with F below d opens down, so p is negative.

H

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ANSWER

15

A parabola has focus $F(1, -1)$ and directrix $y = 1$.

The point $(-3, ?)$ is on it.

Find its y-coordinate.

From V to F is p . That is half of the gap from F to d .

J

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ANSWER

-11

A parabola has focus $F(3, -1)$ and directrix $y = -3$.

The point $(-7, ?)$ is on it.

Find its y-coordinate.

Check the sign of p . It is positive if the curve opens up, negative if down.

K

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ANSWER

–16

A parabola has focus $F(5, 1)$ and directrix $y = 3$.

The point $(-5, ?)$ is on it.

Find its y-coordinate.

Square the whole bracket. The square of a negative number is positive.

L

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ANSWER

–13

A parabola has focus $F(-4, 4)$ and directrix $y = 2$.

The point $(-10, ?)$ is on it.

Find its y-coordinate.

F above the vertex makes p positive, and the curve opens up.

M

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ANSWER

14

A parabola has focus $F(5, 5)$ and directrix $y = 7$.

The point $(-5, ?)$ is on it.

Find its y-coordinate.

The curve opens away from d . Is F above d or below it?

N

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ANSWER

12

A parabola has focus $F(-3, 4)$ and directrix $y = 2$.

The point $(5, ?)$ is on it.

Find its y-coordinate.

Write the vertex as (h, k) before anything else.

R

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ANSWER

-4

A parabola has focus $F(-2, 8)$ and directrix $y = 6$.

The point $(6, ?)$ is on it.

Find its y-coordinate.

Check the answer: its distance to F equals its distance to d .

S

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ANSWER

10

A parabola has focus $F(-2, 2)$ and directrix $y = 0$.

The point $(-6, ?)$ is on it.

Find its y-coordinate.

Measure p from the vertex, which is halfway. Use half the gap.

T

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ANSWER

19

A parabola has focus $F(-2, -1)$ and directrix $y = 1$.

The point $(-10, ?)$ is on it.

Find its y-coordinate.

A point below d is never on a curve that opens up.

U

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ANSWER

-19

A parabola has focus $F(-2, 4)$ and directrix $y = 6$.

The point $(4, ?)$ is on it.

Find its y-coordinate.

Find the vertex first. It is half of the way from F to d .

W

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Scavenger Hunt — where I went

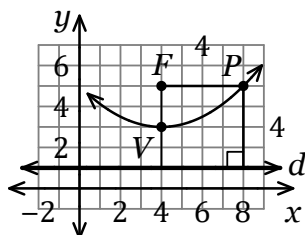
Points on a parabola

Name _____

Period _____

Date _____

Find the y on each card. Walk to the card with your y at the top. A clean run is eight lines. Each card has a line in italics under its question. **If you are on a card because you went wrong, its line says what to check. If you were right, skip it.**



A point is on the parabola when its distance to the focus F equals its distance to the directrix d. **The distance to d** is the perpendicular segment to d. Here d is a line $y = c$, so it is the difference of the y-coordinates.

The vertex V is halfway between F and d, on the perpendicular from F to d. V is on the curve. F is not.

p is the distance from V to F, with a sign. F above V: p is positive and the curve opens up. F below V: p is negative and it opens down. The curve opens away from d.

Steps. 1. Find the vertex V(h, k): halfway between F and d, straight above or below F. 2. Find p: the distance from V to F, positive if F is above V, negative if F is below V. 3. Put x into $(x - h)^2 = 4p(y - k)$ and find y. 4. Check: your point is as far from F as from d.

Example: F(2, 3) and d is $y = -1$, with $x = 6$. Then V(2, 1) and $p = 2$. $(6 - 2)^2 = 8p(y - 1)$, so $16 = 8(y - 1)$ and $y = 3$. Check: (6, 3) is 4 from d and 4 from F.

Card letter	Vertex (h, k)	p	y	Distance to d	Distance to F
1					
2					
3					
4					
5					
6					
7					
8					
9					

A point on a parabola is exactly as far from the focus as from the directrix. How does it feel?

Every number stands for a letter. Write the letter for each answer in the box above its number. Read the boxes you filled, left to right. **If they do not spell the answer, one of your answers is wrong.**

-23 = B -19 = A -16 = L -13 = B -11 = E -10 = N -8 = D -4 = A 5 = N 10 = L 12 = C 14 = A 15 = A 19 = E 23 = D 27 = C

-23	-19	-16	-13	-11	-10	-8	-4	5	10	12	14	15	19	23	27	



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Scavenger Hunt — answers

For the teacher

The ring in order, with the answer found on each card. A group may begin anywhere on it. Wherever it begins, eight right answers bring it back to that card.

B -19 → W -4 → S 23 → C 5 → G 12 → R 19 → U -16 → L -23 → B

A point on a parabola is exactly as far from the focus as from the directrix. How does it feel? The answer is *BALANCED!* It is read along the boxes of the record sheet, which run from the smallest number to the largest.

	Card	What the card gives	Answer, and why	The common wrong answer on this card, and why	Letter of its top
1	B	F(-3, -3), d: y = 1, x = 9.	-19. V(-3, -1) is halfway between F and d, so p = -2. $(9 + 3)^2 = 144 = -8(y + 1)$, so y = -19. Check: (9, -19) is 20 from F and 20 from d.	-10: p is taken as -4, the whole distance from F to d, so 4p becomes -16.	B
2	W	F(-2, 4), d: y = 6, x = 4.	-4. V(-2, 5) is halfway between F and d, so p = -1. $(4 + 2)^2 = 36 = -4(y - 5)$, so y = -4. Check: (4, -4) is 10 from F and 10 from d.	14: p is taken as 1, with the wrong sign, so the curve is turned to open up, toward d.	A
3	S	F(-2, 8), d: y = 6, x = 6.	23. V(-2, 7) is halfway between F and d, so p = 1. $(6 + 2)^2 = 64 = 4(y - 7)$, so y = 23. Check: (6, 23) is 17 from F and 17 from d.	15: p is taken as 2, the whole distance from F to d, so 4p becomes 8.	A
4	C	F(-4, -1), d: y = -5, x = 4.	5. V(-4, -3) is halfway between F and d, so p = 2. $(4 + 4)^2 = 64 = 8(y + 3)$, so y = 5. Check: (4, 5) is 10 from F and 10 from d.	-11: p is taken as -2, with the wrong sign, so the curve is turned to open down, toward d.	D
5	G	F(3, 9), d: y = 7, x = 7.	12. V(3, 8) is halfway between F and d, so p = 1. $(7 - 3)^2 = 16 = 4(y - 8)$, so y = 12. Check: (7, 12) is 5 from F and 5 from d.	10: p is taken as 2, the whole distance from F to d, so 4p becomes 8.	N
6	R	F(-3, 4), d: y = 2, x = 5.	19. V(-3, 3) is halfway between F and d, so p = 1. $(5 + 3)^2 = 64 = 4(y - 3)$, so y = 19. Check: (5, 19) is 17 from F and 17 from d.	-13: p is taken as -1, with the wrong sign, so the curve is turned to open down, toward d.	C
7	U	F(-2, -1), d: y = 1, x = -10.	-16. V(-2, 0) is halfway between F and d, so p = -1. $(-10 + 2)^2 = 64 = -4y$, so y = -16. Check: (-10, -16) is 17 from F and 17 from d.	-8: p is taken as -2, the whole distance from F to d, so 4p becomes -8.	E
8	L	F(5, 1), d: y = 3, x = -5.	-23. V(5, 2) is halfway between F and d, so p = -1. $(-5 - 5)^2 = 100 = -4(y - 2)$, so y = -23. Check: (-5, -23) is 26 from F and 26 from d.	27: p is taken as 1, with the wrong sign, so the curve is turned to open up, toward d.	L

Scavenger Hunt — the cards off the ring

For the teacher

A group reaches these eight cards only by a wrong answer. The number at the top of each one is the wrong answer from one ring card. Its right answer is the number at the top of that ring card, so the group goes to the card it got wrong and tries it again.

Card	What the card gives	Answer, and why	The common wrong answer on this card, and why	Letter of its top
E	F(3, -7), d: y = -3, x = -9.	-23. V(3, -5) is halfway between F and d, so $p = -2$. $(-9 - 3)^2 = 144 = -8(y + 5)$, so $y = -23$. Check: (-9, -23) is 20 from F and 20 from d.	—	N
N	F(5, 5), d: y = 7, x = -5.	-19. V(5, 6) is halfway between F and d, so $p = -1$. $(-5 - 5)^2 = 100 = -4(y - 6)$, so $y = -19$. Check: (-5, -19) is 26 from F and 26 from d.	—	A
J	F(1, -1), d: y = 1, x = -3.	-4. V(1, 0) is halfway between F and d, so $p = -1$. $(-3 - 1)^2 = 16 = -4y$, so $y = -4$. Check: (-3, -4) is 5 from F and 5 from d.	—	A
K	F(3, -1), d: y = -3, x = -7.	23. V(3, -2) is halfway between F and d, so $p = 1$. $(-7 - 3)^2 = 100 = 4(y + 2)$, so $y = 23$. Check: (-7, 23) is 26 from F and 26 from d.	—	E
T	F(-2, 2), d: y = 0, x = -6.	5. V(-2, 1) is halfway between F and d, so $p = 1$. $(-6 + 2)^2 = 16 = 4(y - 1)$, so $y = 5$. Check: (-6, 5) is 5 from F and 5 from d.	—	L
M	F(-4, 4), d: y = 2, x = -10.	12. V(-4, 3) is halfway between F and d, so $p = 1$. $(-10 + 4)^2 = 36 = 4(y - 3)$, so $y = 12$. Check: (-10, 12) is 10 from F and 10 from d.	—	B
A	F(-5, -5), d: y = -7, x = 5.	19. V(-5, -6) is halfway between F and d, so $p = 1$. $(5 + 5)^2 = 100 = 4(y + 6)$, so $y = 19$. Check: (5, 19) is 26 from F and 26 from d.	—	D
H	F(3, -8), d: y = -6, x = 9.	-16. V(3, -7) is halfway between F and d, so $p = -1$. $(9 - 3)^2 = 36 = -4(y + 7)$, so $y = -16$. Check: (9, -16) is 10 from F and 10 from d.	—	C

Other mistakes lead to no card. A group might put the vertex at the focus or on the directrix, write $1/(4p)$ where $4p$ belongs, write $(x + h)^2$ for $(x - h)^2$, make both of the mistakes above at once, or copy a number without its minus sign. Its answer is then not the number at the top of any card, and the group knows to check its work. This sheet calls p the signed distance from the vertex to the focus and writes the equation as $(x - h)^2 = 4p(y - k)$. Some textbooks use other definitions, so tell the class which ones this sheet uses.